

Algebra EOC Study Guide

Functions

Function Notation

$f(x)$ is a fancy way of saying "y".

$f(3)$ – What is "y" when x is 3? Just substitute 3 into the function for the x value and simplify.

Find $f(x)$ from a graph.

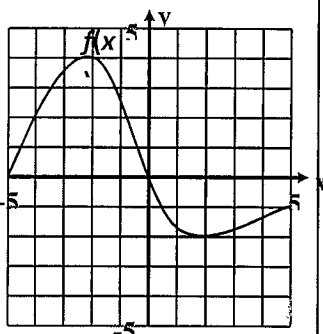
Examples:

Find $f(2)$ in the graph right.

This is asking for the y value when $x = 2$. Therefore, looking at the graph, $y = -2$, then $x = 2$ so $f(x) = -2$.

Find x when $f(x) = -1$.

The x value where $y = -1$ is about .5



Domain and Range (Use Braces {})

Domain – List of x values

Range – List of y Values

Roots – List of x intercepts. (where $y = 0$)

Independent vs. Dependent. Decide which quantity best fits in the sentence ____ (dependent var.) depends on what ____ (independent var.) is.

Time: If there is a time mentioned in the problem it is almost always used as the x value.

Independent Variables – x (horizontal)

Dependent – y (vertical)

Stem and Leaf Plots

An organized chart used to arrange data.

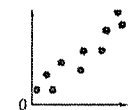
70, 52, 58, 45, 59, 52, 75, 47, 46

4	5 6 7
5	2 2 8 9
6	
7	0 5

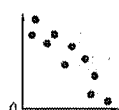
5|2 = 52

- Step 1: Find the least and greatest values
- Step 2: Write the stems in a column
- Step 3: Arrange the leaves from smallest to largest
- Step 4: Write an explanation.

Correlations

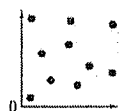


Positive correlation
In general, both sets of data increase together.

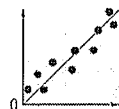
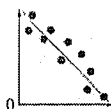


Negative correlation
In general, one set of data decreases as the other set increases.

No correlation
Sometimes data sets are not related.



A trend line on a scatter plot shows a correlation more clearly.



Line of Best Fit vs. Trend Line

Line of best fit is the best trend line.

Finding a Trend Line – Start with identifying the y intercept. Then find the slope and use both to write an equation in slope intercept form with $y = mx + b$ (b = the y intercept) and m as the slope.

Central Tendencies

5, 6, 6, 7, 10, 11, 12, 14, 15, 20

Mode: Most common number (6)

- There may be more than one mode
- There is no mode when all numbers are different

Median: Middle number (10.5)

- List numbers in order
- Find the middle number
- If there are two middle numbers, find the average

Mean: Average (10.6)

- Add all the numbers together
- Divide by the number of items

Range: Difference between the largest number on list and the smallest number (15)

Percentage vs. Flat Changes

Percentage Increases

Measures of Center (Mean, Median & Mode) – All Increase

Measure of Spread (Range) – Increases.

Flat or Constant Increase

Measures of Center (Mean, Median & Mode) – Increase

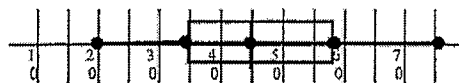
Measure of Spread (Range) – **No Change***.

*because lowest number and highest number raised by same amount so difference stays same.

Box and Whisker Plots

Shows how data is spread out by dividing it into four groups.

20, 36, 58, 45, 59, 55, 75, 35, 35



- Step 1: Draw a line graph with equal intervals
- Step 2: Place a dot on the line graph for the smallest and largest values
- Step 3: Place a dot on the line graph for the median
- Step 4: Place two dots on the line graph, one for the median of the first half of the data and one for the second half
- Step 5: Draw a box around the two middle quartiles
- Step 6: Place an asterisk (*) on the line graph for an extreme data item (if there is one)

Ordering Real Numbers

You may be given numbers in different formats.

Convert them all to decimal form, then order.

Remember that 2.01 is less than 2.1.

Isolating a Single Variable

Express one variable in terms of the others. "solve for" means to isolate a variable by getting rid of everything else. Use same rules as solving an equation – Whatever you do to one side, you do to the other.

Example Solve for t : $\frac{abt}{3w} + 2 = c$

Subtract 2 from both sides for $\frac{abt}{3w} = c - 2$

Multiply both side by $3w$ for $abt = 3w(c - 2)$

Use Distributive Property to get: $abt = 3wc - 6w$

Divide both sides by ab to get $t = \frac{3wc - 6w}{ab}$

Algebra EOC Study Guide

Parent Functions & Transformations

$$y = mx + b \rightarrow \text{Linear}$$

$$y = a|x - h| + k \rightarrow \text{Absolute Value}$$

$$y = a^x \rightarrow \text{Exponential}$$

$$y = a(x - h)^2 + k \rightarrow \text{Quadratic}$$

$$y = \frac{a}{x - h} + k \rightarrow \text{Rational}$$

$$y = a\sqrt{x} \rightarrow \text{Square Root (Radical)}$$

h: Horizontal Shift; k: vertical shift; a: stretch; m: slope

A negative before the a indicates upside down direction

Absolute Value Transformations

move up (a positive number outside the A.V. bars).

move down (a negative number outside the A.V. bars)

move left (a positive number inside the A.V. bars)

move right (a negative number inside the A.V. bars)

$$y = |x - 2| + 3 \text{ Right 2, Up 3}$$

$$y = |x + 5| - 6 \text{ Left 5, Down 6}$$

Exponential Functions

$$y = ab^x$$

a = Beginning amount (if = 1, then invisible)

b = Rate or multiplier

x = Periods that multiplier is used.

Remember that a percentage increase or decrease should be added or subtracted from 1.

a 5% increase = Rate of 1.05;

1.2 decrease = Rate of .988)

Word Problems

"DERT" Formula: Distance = Rate x Time

Consecutive, means no skipping between numbers.

Look for a starting or beginning number and use that as your constant or y-intercept.

Look for words like ratio or per to know that you have a rate and use it as a coefficient for your x.

Remember that if you are using something up it will be a negative slope.

Solving "4:5:9" Ratio Type Problems

Example: *Three angles in a triangle have the ratio 4:5:9. What is the measure of each angle?*

Method #1: Add up the numbers and use as a denominator.
In the example, the 3 numbers have a sum of 18.
Use the 18 as a denominator and each number as the numerator.
Multiply by the total to get each number.

Sequences

State Description of possible questions

Standards Description from OSP

Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, ...

What is the slope of the line that contains the points associated with these values and their position in the sequence?

How is the slope of the line related to the sequence?

• Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n

is a positive integer,

a. find $u(5)$;

b. find n so that $u(n) = 367$; and

c. find a formula for $u(n)$.

• Write a recursive formula for the geometric sequence 5, 10, 20, 40, ... and determine the 100th term.

• Given that $u(0) = 2$ and $u(n + 1) = 3u(n)$,

a. find $u(4)$, and

b. find a formula for $u(n)$.

*Note – This was changed from 361 on state website which does not generate an integer answer.

Answer

$$a_n = a_{n-1} + 4$$

4

Slope & d (common difference) are the same.

If $u(0) = 3$, then 3 is $a_1 - d$
 $u(n+1) = u(n) + 7$, then $d=7$

Therefore $a_1 = 10$

$$a. u(5) = 10 + (7 \cdot 4) = 38$$

$$b. 367 = 10 + 7(n-1)$$

$$367 = 10 + 7n - 7$$

$$367 = 3 + 7n$$

$$364 = 7n$$

$$n = 52$$

$$c. a_n = 10 + 7(n - 1)$$

$$r = 3, a_1 = 6 \text{ (from } 2 \times 6)$$

$$a. u(4) = 6 \cdot 3^3$$

$$u(4) = 162$$

$$b. a_n = 6 \cdot r^{n-1}$$

Notes on Sequences

Arithmetic sequences – (Adding a common difference)

d = common difference (# added)

To find d: Subtract 1st term from 2nd.

Recursive Form for nth term/ Uses the prior term (a_{n-1})

$$a_n = a_{n-1} + d$$

Explicit Form or nth term/ does not use prior term.

$$a_n = a_1 + d(n - 1)$$

a_1 = the 1st term

d = common difference

d = the slope of a line

To find d: Subtract 1st term from 2nd

Geometric Sequences (Multiplying by a common rate)

Recursive Form for nth term/ Uses the prior term (a_{n-1})

$$a_n = a_{n-1} \cdot r$$

Explicit Form or nth term/ does not use prior term.

$$a_n = a_1 \cdot r^{n-1}$$

a_1 = the 1st term

r = common ratio

To find r: Divide 2nd term by 1st.

r = common ratio (# multiplied)

Order of Operations

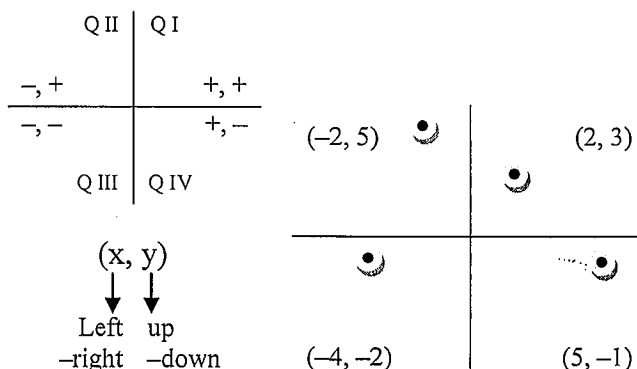
1. Parentheses: inside out $[], \{\}, \dots$
2. Exponents
3. Multiply or divide: Left \rightarrow Right
4. Add or subtract Left: \rightarrow Right

Division Bar

Examples

$$\begin{array}{rcl} 3 - 12 \div 4 & 6^2 \div 4 \cdot 3 & 4^2 \cdot 2 + [7 - (3^2 - 5)] \\ 3 - 3 & 36 \div 4 \cdot 3 & 16 \cdot 2 + [7 - (3^2 - 5)] \\ 0 & 9 \cdot 3 & 32 + [7 - 4] \\ & & 35 \end{array}$$

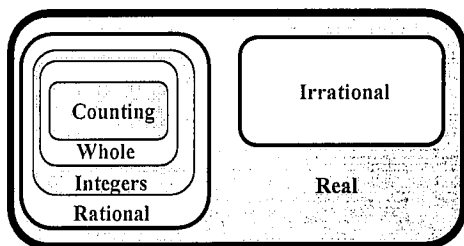
Coordinate Graphing



Real Numbers

- **Natural or Counting Numbers:** $N \in \{1, 2, 3, 4, \dots\}$
- **Whole Numbers:** $W \in \{0, 1, 2, 3, 4, \dots\}$
- **Integers:** $I \in \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- **Rational Numbers (R):** Any number that can be written as a fraction a/b . *Natural, whole, and integers are rational.*
- **Irrational Number (Q):** Numbers that CANNOT be written as a fraction, like non-repeating decimals and radicals
- **Real Numbers (R):** All rational and irrational numbers

Numbers are either rational or irrational



Integers

How to Add Integers

- **Same signs:** Add and keep the sign
- **Different signs:** Subtract large - small
Keep sign of the larger number

How to Subtract Integers

- Change all subtraction problems to addition
 $a - b = a + (-b)$ $a - (-b) = a + b$
- Follow rules for addition of integers

How to Multiply and Divide Integers

- **Two Numbers**
Same signs \rightarrow Positive
Different signs \rightarrow Negative
- **More than two Numbers**
Odd \rightarrow Negative
Even \rightarrow Positive

Properties

- **Commutative Property of Addition/Multiplication**
 $a + b = b + a$ $ab = ba$
- **Associative Property of Addition/Multiplication**
 $(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
- **Distributive Property**
 $a(b + c) = ab + ac$
- **Identity Properties**
 $a + 0 = a$ and $0 + a = a$ $a \cdot 1 = a$ and $1 \cdot a = a$
- **Inverse Properties**
 $a + -a = 0$ $a \cdot 1/a = 1$

The number $1/a$ is called the reciprocal or multiplicative inverse.

- **Properties of 0**
 $a \cdot 0 = 0$ and $0 \cdot a = 0$
 $0/a = 0$ where $a \neq 0$
Division by 0 is undefined

Add-Subtract-Multiply-Divide Polynomials

Adding and Subtracting

- Combine like terms only
- Rewrite subtraction problems
- Watch SIGNS!

Examples

$$\begin{array}{r} (3x + 4) + (2x - 1) \\ 5x + 3 \end{array}$$

$$\begin{array}{r} (4d - 2) - (5d - 3) \\ 4d - 2 - 5d + 3 \\ -d + 1 \end{array}$$

$$\begin{array}{r} (x^2 + x + 1) + (2x^2 + 3x + 2) \\ 3x^2 + 4x + 3 \end{array}$$

$$\begin{array}{r} (4x^2 + 3x + 2) - (2x^2 - 3x + 7) \\ 4x^2 + 3x + 2 - 2x^2 + 3x - 7 \\ 2x^2 + 6x - 5 \end{array}$$

$$2x(5x - 4) = 10x^2 - 8x$$

$$-2(5a - 4) = -10a + 8$$

$$\frac{2x^2 + 6}{2} = x^2 + 3$$

$$\frac{20c + 8}{4} = 5c + 2$$

Equations

$$3x = 27 \Rightarrow \text{Divide for answer} \Rightarrow x = 9$$

$$\frac{x}{3} = 18 \Rightarrow \text{Multiply for answer} \Rightarrow x = 54$$

Two-Step Equations

Solve in reverse order of operations

- $+/-$ first
- then x/ \div

Watch for negatives

$$\begin{array}{rcl} -7 - 13y & = & 32 \\ -13y & = & 39 \\ y & = & -3 \end{array}$$

$$\begin{array}{rcl} \frac{x}{4} + 5 & = & 12 \\ \frac{x}{4} & = & 7 \\ x & = & 28 \end{array}$$

Variables on Both Sides of Equation

Move the smaller variable by adding the opposite to both sides of the equation. (look to move negative variables)

$$\begin{array}{rcl} (+3y) 5y - 10 & = & 14 - 3y (+3y) \\ 8y - 10 & = & 14 \\ 8y & = & 24 \\ y & = & 3 \end{array}$$

More Equations

Combine Like Variables

$$\begin{array}{rcl} 4n - 2 + 7n & = & 20 \\ 11n - 2 & = & 20 \\ 11n & = & 22 \\ n & = & 2 \end{array}$$

Distribute to Eliminate Parentheses

$$\begin{array}{rcl} 3(r - 4) & = & 9 \\ 3r - 12 & = & 9 \\ 3r & = & 21 \\ r & = & 7 \end{array}$$

Multi-Step Equations

1. Distribute
2. Combine
3. Move variables to the same side of the equation
4. Solve remaining equation

*Big Tip: Look to get rid of fractions on variables.
Multiply all terms by the common denominator.*

More Equations

Multi-Step Example

$$\begin{array}{rcl} 3x - 2(x + 6) & = & 4x - (x - 10) \\ 3x - 2x - 12 & = & 4x - x + 10 \\ x - 12 & = & 3x + 10 \\ -12 & = & 2x + 10 \\ -22 & = & 2x \\ -11 & = & x \end{array}$$

Literal Equations

Treat variables like numbers...

$$\begin{array}{l} T + M = R, \text{ solve for } T \\ T = R - M \end{array}$$

$$\begin{array}{l} A = lw, \text{ solve for } w \\ \frac{A}{l} = w \end{array}$$

$$\begin{array}{l} ax + r = 7, \text{ solve for } x \\ ax = 7 - r \\ x = \frac{7 - r}{a} \end{array}$$

Proportions

Proportion: An equation showing that two ratios are equal.

Cross Products are equal

$$\frac{a}{b} = \frac{c}{d} \quad a \cdot d = b \cdot c$$

$$\frac{3}{4} = \frac{9}{12} \quad 4 \cdot 9 = 3 \cdot 12$$

$$36 = 36$$

Examples

$$\frac{15}{m} = \frac{10}{9} \quad 10m = 135 \quad m = 13.5$$

$$\frac{9}{27} = \frac{25}{x}$$

$$9x = 675$$

$$x = 75$$

Percent Problems

Converting Percents

- **P \Rightarrow D:** Move decimal two to the left (2 \leftarrow)
- **D \Rightarrow P:** Move decimal two to the right (2 \rightarrow)
- **P \Rightarrow F:** Write number over 100 and reduce
Hint: Write percents with one decimal over 1000, two decimals of 10,000, etc.
- **F \Rightarrow D:** Divide numerator by denominator

Percent Problems using Equations

- Write the equation as you read the problem.
is \rightarrow = of \rightarrow x what \rightarrow x into, out of $\rightarrow \div$
- Write percents as decimals or fraction

Percent Change

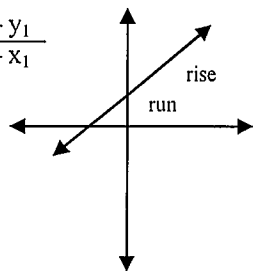
$$\text{Percent} = \frac{\text{change}}{\text{original}} \quad \text{Subtract amounts and divide by original}$$

Test Taking Tips

- Skip problems that you don't know how to do and come back.
- Write down all your steps. In other words, don't do the work in your head.
- When you panic, stop, and do something that relaxes you, like close your eyes and take a deep breath.
- If you finish early, go back and do the problems again. Don't look at your previous work.

Slope

Slope Formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$



/ \Rightarrow Positive Slope

\ \Rightarrow Negative Slope

— $\Rightarrow m = 0$ ($y = 8$)

| \Rightarrow Undefined Slope ($x = 8$)

Parallel Lines: Same slope

Perpendicular Lines: Reciprocal opposite of the given line

$$m = 3 \quad \perp m = -\frac{1}{3}$$

Examples

$$(5, 3); (4, -1)$$

$$(4, 8); (4, 2)$$

$$3 - (-1)$$

$$8 - 2$$

$$5 - 4$$

$$4 - 4$$

$$m = 4$$

m is undefined

Slope Intercept Form

Slope-Intercept Form:

$$y = mx + b$$

y-intercept,
where the line
crosses the y-

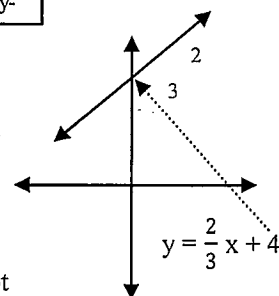
slope

x and y intercepts:

- x: Substitute 0 for y and solve for x
- y: b is the y-intercept

Finding an Equation:

- Find the slope
- Use one point to find the y-intercept



Finding a Line Examples

$$m = 3; (2, 6)$$

$$(1, 8); (4, 2)$$

$$y = mx + b$$

$$y = mx + b$$

$$6 = (3)(2) + b$$

$$2 = (-2)(4) + b$$

$$6 = 6 + b$$

$$2 = -8 + b$$

$$b = 0$$

$$10 = b$$

$$y = 3x + 0$$

$$y = -2x + 10$$

$$y = 3x$$

$$m = \frac{2-8}{4-1} = -2$$

Standard Form

Standard Form of a Linear Equation

- $Ax + By = C$
- $m = -\frac{A}{B}$
- The x will always be positive
- No fractions: multiply by common denominator

Examples

$$7y = -5x - 35$$

$$x = \frac{2}{3}y + 6$$

$$5x + 7y = -35$$

$$3x = 2y + 18$$

$$3x - 2y = 18$$

Solving Inequalities

Graphing Symbols

- Open: $<$ and $>$ ○
- Closed: \leq and \geq ●

When dividing and multiplying by a negative number, **REVERSE** the inequality.

Examples

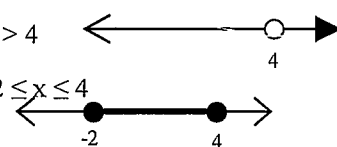
$$15 - \frac{t}{4} > 10$$

$$x > 4$$

$$-\frac{t}{4} > -5$$

$$t < 20$$

$$-2 \leq x \leq 4$$



Absolute Value Equations/Inequalities

Equations: Solve twice using + AND -

Inequalities: Solve twice using + AND - and reverse the inequality

Examples

$$|3x - 2| = 10$$

$$|x - 6| > 2$$

$$3x - 2 = 10 \text{ OR } 3x - 2 = -10$$

$$x - 6 > 2 \text{ OR } x - 6 < -2$$

$$3x = 12 \text{ OR } 3x = -8$$

$$x > 8 \text{ OR } x < 4$$

$$x = 4 \text{ OR } x = -\frac{8}{3}$$

Reverse inequality;
Negative added
to the 2

Substitution/Elimination

Substitution Method: Use when you can easily find the value of either the x or y variable.

$$15x - 5y = 30$$

$$15x - 5(2x + 3) = 30$$

$$y = (2)(9) + 3$$

$$y = 2x + 3$$

$$15x - 10x - 15 = 30$$

$$y = 21$$

$$5x - 15 = 30$$

$$5x = 45$$

$$\text{Answer: } (9, 21)$$

$$x = 9$$

Elimination Method: Use when there is no easy way to substitute. Multiply one or both equations by a corresponding factor. (Use opposites so you can add equations together.)

$$2x + 3y = 1$$

$$10x + 15y = 5$$

$$2x + (3)(-1) = 1$$

$$5x + 7y = 3$$

$$-10x - 14y = -6$$

$$2x - 3 = 1$$

$$y = -1$$

$$2x = 4$$

$$x = 2$$

$$\text{Answer: } (2, -1)$$

Consistent/Inconsistent Systems

Inconsistent System: No solution, Lines are parallel

$$\begin{aligned} 2x - y &= -1 \\ 4x - 2y &= 4 \\ 0 &\neq -6 \end{aligned}$$

Both lines have the same slope

Consistent System:

- Independent: lines intersect, One solution
- Dependent: Infinitely many solutions, Same line!

$$\begin{aligned} x - y &= 2 \\ 2x - 2y &= 4 \\ 0 &= 0 \end{aligned}$$

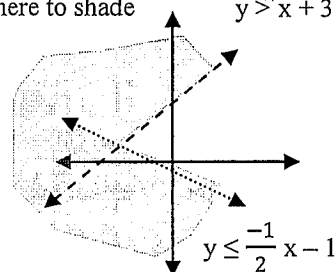
Second equation is 2x the first equation

Graphing Inequality Systems

$>$ $<$ \leftarrow \rightarrow Type of line

\geq \leq \leftarrow \rightarrow Type of line

$>$ $<$ above below Where to shade



Coin Problems/Age Problems

Coins

Usually involved with number of coins and monetary value

A coin bank has 250 dimes and quarters worth \$39.25. How many dimes and quarters are there?

$$d + q = 250 \quad .10d + .25q = 39.25$$

Age

Represent a person's age in the past, present, or future

A father is 32 years older than his son. In four years, the father will be 5 times older. How old are they now?

$$\begin{aligned} s + 32 &= f \leftarrow (\text{age now}) \\ f + 4 &= s + 4 \leftarrow (\text{age in 4 years}) \\ f + 4 &= 5(s + 4) \end{aligned}$$

Wind/Current and Digit Problems

Wind/Current

Need speed when traveling with the wind and against the wind

$$\text{Use } r \cdot t = d$$

$$(r + w)2.5 = 750$$

$$(r - w)2 = 750$$

r = rate; w = wind

Digit

Write value of number in expanded form (t: tens; u: ones)

- Two digit number: $10t + u$
- Reverse digits: $10u + t$
- Sum of digit: $t + u$

Laws of Exponents

Product of Powers: $x^m \cdot x^n = x^{m+n}$ $x^6 \cdot x^4 = x^{10}$

Power of Power Property: $(x^m)^n = x^{m \cdot n}$ $(x^6)^4 = x^{24}$

Power of a Product: $(xy)^m = x^m y^m$ $(xy)^5 = x^5 y^5$

Hint: even powers \rightarrow Positive answer
odd powers \rightarrow Negative answer

$$m > n \quad \frac{x^m}{x^n} = x^{m-n}$$

$$\frac{x^6}{x^4} = x^2$$

$$m < n \quad \frac{x^m}{x^n} = \frac{1}{x^{n-m}}$$

$$\frac{x^4}{x^6} = \frac{1}{x^2}$$

Power of Fractions:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\left(\frac{x}{y}\right)^6 = \frac{x^6}{y^6}$$

$$x^{-n} = \frac{1}{x^n}$$

$$x^{-6} = \frac{1}{x^6}$$

$$x^0 = 1$$

$$5^0 = 1$$

Scientific Notation

A number written as a product with two factors:

- A number between 0 and 9 and
- A power of 10

Examples

$$463,000,000 = 4.63 \times 10^8$$

$$.000597 = 5.97 \times 10^{-4}$$

Move right for a negative exponent; left for positive

Count the number of times you need to move the decimal to make a number less than 10

$$(3 \times 10^3)(4 \times 10^{-5})$$

$$12 \times 10^{-2}$$

$$1.2 \times 10^{-1}$$

$$\frac{2.5 \times 10^6}{5 \times 10^2}$$

$$.5 \times 10^4$$

$$5 \times 10^3$$

Add exponents

Subtract exponents